Team Control Number

For office use only
T1 ________________
T2 ________________
T3 ________________
T4 ________________

41503

Problem Chosen

B

For office use only
F1 ________________
F2 ________________
F3 ________________
F4 ________________

2015 Mathematical Contest in Modeling (MCM) Summary Sheet
(Attach a copy of this page to your solution paper.)

Type a summary of your results on this page. Do not include
the name of your school, advisor, or team members on this page.

Summary

We formulate a searching model about two parts: how to confirm the position of the wreckage of the plane and how to send the search and rescue forces reasonably.

The first part, we first determine a large search range, narrowing the final searching range with limit method. Then divide the searching range, taking one of the regions as example, we consider the searching route in one area. For the search in the region, we use the Hamilton circuit to get the search route, and then make an analogy to the search of the whole region.

The second part, we use “lingo” software and principle related to Operations research to obtain the optimal dispatch scheme. After querying and summarizing the searching ability of different types of ships and aircraft, we design a program. Through testing, calculating, comparing the test data, we obtain the best search and rescue dispatch scheme.

There are still some unsolved problems in the model, so we also have put forward a preliminary solution as our future work direction.

Key words:

Hamilton circle; “Lingo” software; the search for wreckage of plane; optimal search scheme

Where is the missing aircraft wreckage?

Summary

We formulate a searching model about two parts: how to confirm the position of the wreckage of the plane and how to send the search and rescue forces reasonably.

The first part, we first determine a large search range, narrowing the final searching range with limit method. Then divide the searching range, taking one of the regions as example, we consider the searching route in one area. For the search in the region, we use the Hamilton circuit to get the search route, and then make an analogy to the search of the whole region.

The second part, we use "lingo" software and principle related to Operations research to obtain the optimal dispatch scheme. After querying and summarizing the searching ability of different types of ships and aircraft, we design a program. Through testing, calculating, comparing the test data, we obtain the best search and rescue dispatch scheme.

There are still some unsolved problems in the model, so we also have put forward a preliminary solution as our future work direction.

Key words:

Hamilton circle; "Lingo" software; the search for wreckage of plane; optimal search scheme

Contents

1. Introduction and background

In March 8, 2014, MH370 from Kuala Lumpur International Airport to Beijing Capital International Airport lost contact with air traffic control in the Malaysia and Vietnam radar coverage boundary. After the crash was confirmed, the wreckage of the plane has not been found so far. According to the statistics, the global average annual have been 27 large commercial aircraft accidents will happen in average all over the world every year and hundreds of thousands of people will die in the air crash.

When the disaster occurs, only the accurate and rapid determination of the location of the plane crash and the wreckage, it is possible to implement the rescue to the maximum extent. However, finding the location of the plane crash will be confronted with many difficulties: the lost connection of the plane, whether the aircraft exploded, different signal sources from different plane, the increase of the scope of the search especially when the plane crashed into the seas, no signal from the black box, ocean currents which are unknown in the ocean, whether the plane fell into the deep canyon, the limited search and rescue forces and so on. All the factors mentioned above will hinder or delay the confirmation of wreckage.

Now we take MH370 as an example, hope to be able to build a model, thus helping us determine the approximate range of the crashed plane, at the same time, get the optimum number of ships and aircraft to make our search and rescue achieve the highest efficiency and the least expensive.

2.Problem analysis

2.1How to determine the searching scope of the wreckage

In order to get the rough search range of the wreckage, first we must determine the last received signal of the position and the height of the flight, in order to judge the amount of fuel remaining in the aircraft at that time. Therefore we can calculate the maximum distance the plane can fly after that. Estimate the sea range where the aircraft crashed according to the free fall. Then determine rough position range of the wreckage of the plane at the bottom of the sea judging by the searching ability of the planes or ships, the theoretical searching scope and the depth of the sea.

2.2The search of the wreckage in the determined scope

According to our analysis mentioned above, once the range of aircraft crash is identified, what we have to do is to use the resources we have found within the stipulated time to do the effective and fast search. To achieve the efficiency of search, we will divide the searching area into a plurality of blocks, then, take one of these as example, from Hamilton loop principle, consider the search route in the diamond and make sure the lowest amount of paths around the corner. At last we can get the shortest searching route.

2.3How to dispatch the searching power to achieve the optimization

When the final rescue route is determined, what we need to consider is how to send our search and rescue forces. Because the search radius of different search aircraft and ships are different, the cost per hour are different, then sending how many boats and aircraft to achieve maximum utilization is the most critical, here we will use "lingo" software and linear programming method of operational research learning to get the answer.

3.Assumptions

We make the following assumptions in this paper.

- Assuming the fall of the plane is completely free fall motion
- Assuming the falling path of the aircraft is a parabola
- The hypothesis of the seabed is a plane and the depth of the sea is fixed
- The search is not affected by the weather, equipment continuous working
- Do not consider the underwater currents motion

4.Terminology definitions

R is the maximum mileage the aircraft can fly after losing contact

H is the height of the aircraft above the ground of sea

V is the speed of the aircraft

g is gravity

t is the time for rescue

S is the scope of the search

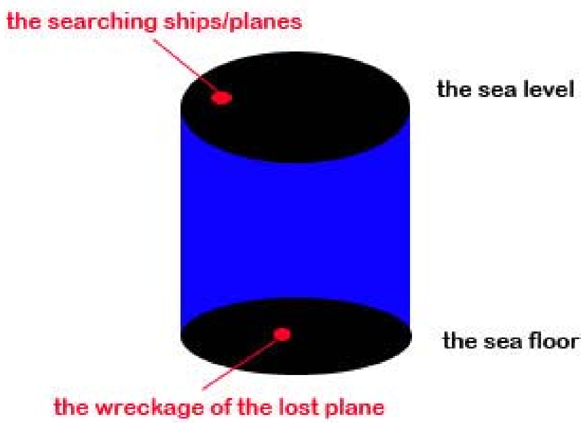
5. Models

5.1Searching model

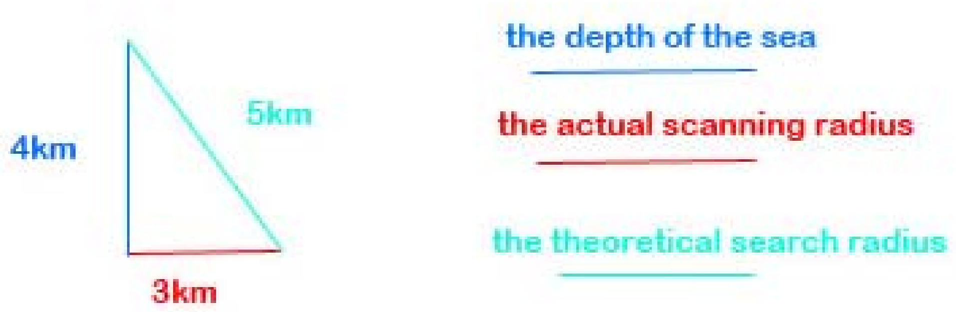
5.1.1Search for the modeling and analysis

The volume of the wreckage of the plane is extremely small compared with the searching region, so we regard it as a particle. Because R and H are fixed, we regard the searching region as a circle whose radius is S. And then we can get a formula:

$S = \pi \cdot [R + V \times (\sqrt{2H / g})]^2$。



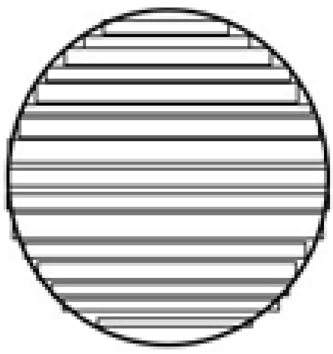
As is shown in the picture, when the scope of search is confirmed, we extend it to the bottom of the sea to get a cylinder whose bottom surface is S, this is the searching range we have determined at first.



In fact, aircraft and ships searching ability in practice cannot achieve the theoretical value. Assuming that the sea bottom depth is 4km, the theoretical search radius is 5km, but by the Pythagorean Theorem we know that the actual scanning radius is 3km. So the following ability of search has already been handled through the Pythagorean Theorem.

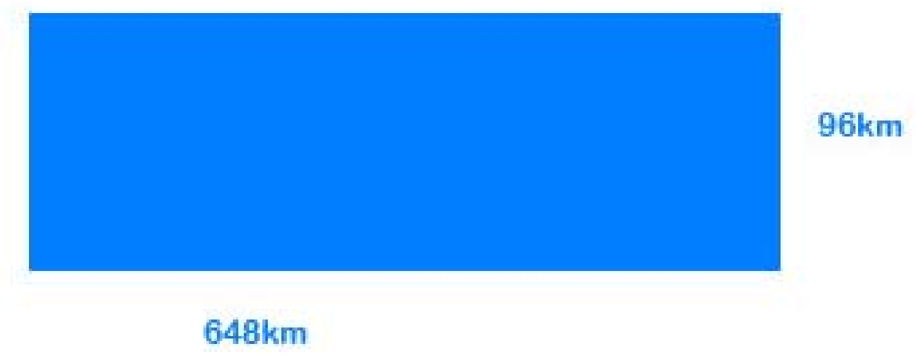
5.1.2Establishment and processing of model

We divide the submarine circle searching area into many diamonds (when the segmentation is infinitely small, the sum area of these small diamonds is equal to the area of the circle). As is shown in picture:

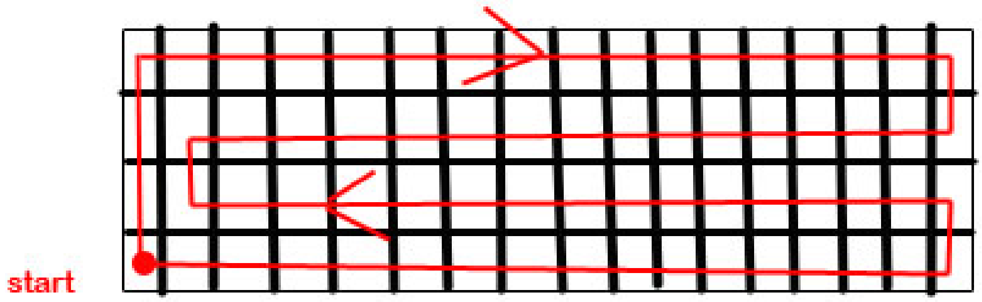


Every diamond will be arranged with rescue power to carry on searching, according to the size of the diamond. We take one from these divided diamonds, and then set it into a square zone whose size is 648km length and 96km width, as well as the actual searching radius which is

4km. And each square zone should be assigned with ships to search. The pictures are as follows:



As we know that searching zone and speed of ships are fixed, we should expand the searching radius and reduce searching distance so as to make the time we used for searching as little as possible. As for expanding searching radius, we entertain that we can line up three boats whose distance from each other is 8km.Thus we can make a 24km-wide transverse scanning area which covers the biggest area in a time unit. Next, we may talk about reducing the distance. According to the lining style of the three ships, we can divide the searching zone into 108 24×24 small diamonds. Now, we are faced with the problem that how to cover every small diamonds in the least time. Owing to the fixed speed, we just need to make the route as short as possible. Applying the knowledge of the picture theory, we can consider every small diamond as a point, and then the shortest searching route can be turned into the aim to figure out the smallest Hamilton circle which passes all of the points. But considering that all the small diamonds are the same in size, so we find that all the Manhattan circle are also the same in size, namely the same in distance. Through further analysis and finding related information，we learn that in order to use the shortest time, the numbers of the corners in the searching route should be reduced, which requires searching ships to go straight lines as much as possible. In the light of these principles, we get the final searching route picture.



5.1.3Strength and weakness

Strength：

- Use the Hamilton circuit principle to make the rendering of the PS
- To get the actual searching radius

Weakness：

- This searching model doesn't consider the current ocean in the bottom of the sea
- Flotilla may be unable to scan comprehensively when meet the corner area
- Consider the seabed as ideal plane inside of taking the actual condition in account

5.2The best searching plan

5.2.1The ideas about how to reduce the cost of model

We suppose that ship and plan are our only existing power, and planes and ships are dispatched by the same point. As the searching area is fixed, we can consider it as Constraint variable, and build the linear programming model aims at the total cost so as to achieve the least cost.

Referring to the relevant materials, we suppose the information of the planes and ships' searching power.

Ship number	Searching ability
1	20m^2/h
2	48 m^2/h
3	51 m^2/h
4	78 m^2/h
5	104 m^2/h
6	232 m^2/h
7	228 m^2/h
8	250 m^2/h
9	299 m^2/h

Plane number	Searching ability
1	280 m^2/h
2	292 m^2/h
3	320 m^2/h
4	312 m^2/h
5	409 m^2/h
6	388 m^2/h
7	480 m^2/h

5.2.2Establishment and processing of the model

Suppose that we can use up to 30 ships, 20 aircrafts and must solve the problem in 30 days.

In addition, we get the fuel consumption of ships and planes by survey data and take factors such as artificial cost into account. Then we draw a conclusion that ships need to spend" a=3000 Yuan" per hour for search, the planes need "b=10000 Yuan".

To solve the problem conveniently, we take the average among the search and rescue capabilities of various types of ships and planes as the standard. Get that the ability to search of the ships: m1=（20+48+51+78+104+232+228+250+299）/9=145.5556m^2/h

The ability to search of the planes:m2=(280+292+320+312+409+388+480)/7=353.4286m^2/h

Supposing that we need "x" ships, "y" planes to search "Km^2" and cost "z".

Then t=k/(m1*x+m2*y)。

Minz=(a*x+b*y)*t

0≤x≤30

0≤y≤20

0≤t≤30*24

Using "lingo" software to solve the problem, we can get the best search and rescue plan is to use 9 ships and no plane.

5.2.3Strength and weakness

Strength：

- Use "lingo" software to obtain the best solution
- Provide the best and the most actual dispatch scheme

Weakness：

- Only consider two search and rescue forces: the aircraft and ships, without taking into account the satellite.
- The rescue forces of ships and planes we have considered is not complete
- Without considering which country is the nearest to the wreckage of the plane, so we don't know ask which country to send the rescue forces.

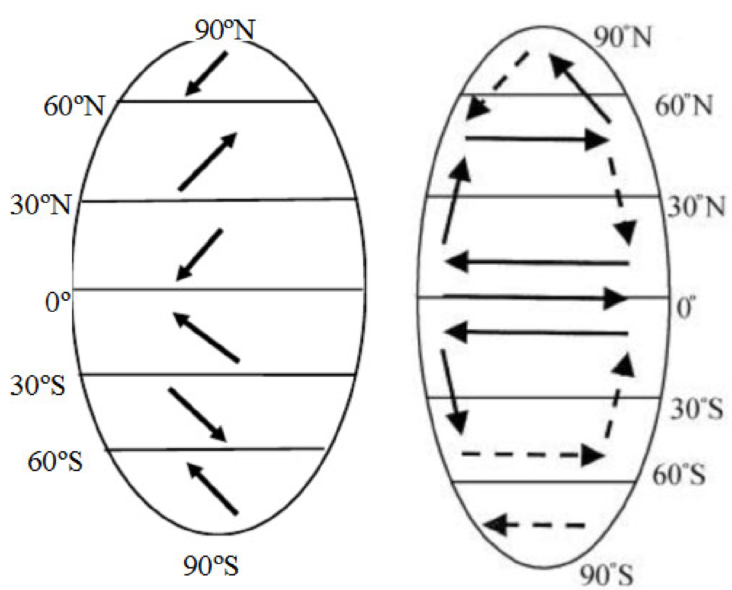
6. Future work

The effect of ocean currents to determine the wreckage

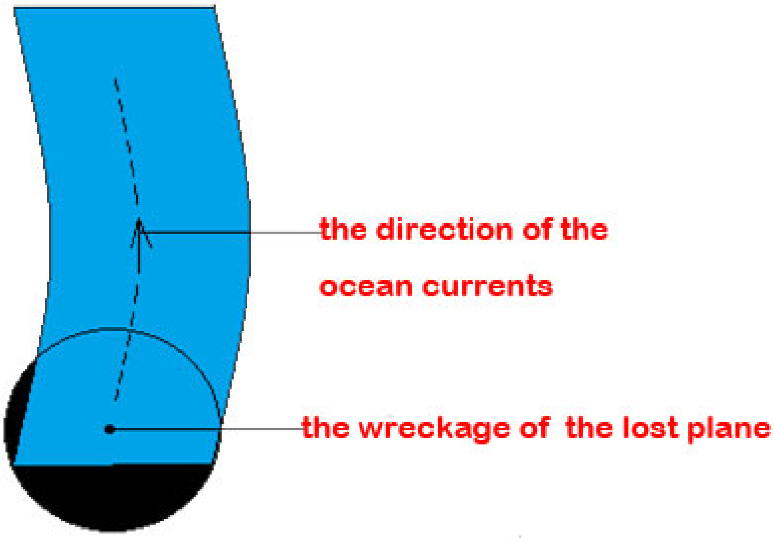
When we determine the falling range of the wreckage in the sea, we haven't considered the condition of the sea before in the original model. Thus, in order to improve the model, what we also need to do is to judge the impact of ocean currents on searching the wreckage.

Assume that the distribution of the ocean currents is in ideal state, as is shown in the picture:

the distribution of ocean currents



Then, among the four places where plane may fall: the Arctic Ocean, the Atlantic Ocean, India Ocean, the Pacific Ocean, except that the Arctic Ocean does counterclockwise motion according to the ideal, the ocean currents of the remaining three oceans can be divided according to their position: south or north. Searching some datum we can know that the direction of the ocean currents in the northern hemisphere is clockwise while in the southern hemisphere is counterclockwise.



Symbols :

v_1:the minimum speed of the wreckage of the plane with the ocean currents movement (m/h)

v_2:the maximum speed of the wreckage of the plane with the ocean current movement (m/h)

T: the time which has passed since the plane crash happened (hours)

Because of the ocean currents movement, the wreckage of the plane will also move, which may make the wreckage leave the original circle search scope we have booked. Due to the quality of the wreckage, the blocking behavior of the seabed sediment and many other factors, we will extend the circle search scope to strip search area. According to the ocean currents speed and the rough time when the plane crash happened, we can calculate the movement distance of the planes' wreckage at the bottom of the sea. Assuming the maximum and minimum speed which the wreckage of the plane has with ocean currents in the bottom of the sea, draw a general strip search area. The strip search region can be approximately regarded as a rectangle, so we can identify a more precise search range: ($v_2t - v_1t$) $\cdot R$

7. References

[1]

"the best rescue route problem based on Graph theory and nonlinear programming", Henan University of Science and Technology

[2] *"Operations research (Fourth Edition)",Tsinghua University*

[3] *"Lingo" optimal modeling software, Xie Jinxing Xue Yi, Tsinghua University press*

8. The speech of news conference

Our airline company wants to express our deep condolences to the people died in the air crash.

The flight from A to fly to B has unfortunately crashed. In the first moment of the accident, we immediately launched search and rescue, and reported the matter to the world. In the next three days, the surrounding countries all do their best to send their search and rescue teams. We express our sincere thanks to their help. They have sent 34 aircraft and more than 40 ships constantly, thus the searching range is expanding. But unfortunately the wreckage of the crashed airplane is still unfound.

So far the search has been carried out for nearly a week, we still have not given up the search for the wreckage of the crashed airplane, and now we have some specific arrangements and plans about the search and rescue for the next six months.

According to the newest search results, we can determine that the plane has plunged into the sea. We get the flight height and distance according to the position of signal before the last connecting moment. Based on these data, we get the residual fuel of the plane, calculate the maximum miles it may fly and judge that the falling area of the crashed aircraft is the North Pacific Ocean. We will regard the search range as a ball whose center is the position when the fuel is used up, whose radius is the maximum miles the plane finally flies and the columnar when the ball falls into the sea. We make use of the working principle of searching tool such as "underwater sonar" to precisely confirm the searching range for the convenience of carrying out our underwater searching equipment.

After determining the search range underwater, we will send ships and planes for further search. We will divide the search space into some rectangular boxes, using the Hamiltonian circuit principle, carrying out back and forth search to these small areas, and eventually we find the wreckage of the plane. When sending out search power, taking into account the searching efficiency of ships and aircraft, we use the "lingo" software to calculate and obtain the best dispatch scheme, make our search efficiency to achieve the maximum, and have some emergency power to cope with some possible emergencies.

To make a long story short, the model used by the search for our future is a combination of satellites, ships, aircraft, underwater sonar and other aspects of the force, making the best use of all the resources find the wreckage of the plane. After that we will do a comprehensive and detailed analysis on the causes of the accident, and truthfully report to you. We will take this as a warning, conscientiously learn from the accident, so that our future work will be more perfect, and make our flying more safety and security.

Finally we want to express our regret and condolences to the happening of this accident again and show thanks to countries which carry out humanitarian, giving our help in this search. We will continue working to improve our search scheme, and continue to provide the newest search results to everyone.

ENDS.

9. Appendix

Use “lingo” software to solve the problem, the procedure is as follows:

```
min=(3000*x+10000*y)*180000/(145.5556*x+354.4286*y);
x<=30;
y<=20;
180000/(145.5556*x+354.4286*y)<=720;
@gin(x);
@gin(y);
```

The result of its operation is as follows:

Local optimal solution found.

Objective value:	3709923.
Objective bound:	3709923.
Infeasibilities:	0.000000
Extended solver steps:	5
Total solver iterations:	260

Variable	Value	Reduced Cost
X	9.000000	0.000000
Y	0.000000	370303.2

Row	Slack or Surplus	Dual Price
1	3709923.	-1.000000
2	21.00000	0.000000
3	20.00000	0.000000
4	582.5955	0.000000